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POISSON-LIE T-DUALITY IN N=2 SUPERCONFORMAL FIELD THEORIES.

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Abstract

The supersymmetric generalization of Poisson-Lie T-duality in superconformal WZNW models is considered. It is shown that the classical N=2 superconformal WZNW models possess a natural Poisson-Lie symmetry which allows to construct dual σ - models.

Introduction.

Target space duality in string theory has attracted a considerable attention in recent years because it sheds some light on the geometry and symmetries of string theory. The well known example of T-duality is mirror symmetry in the Calaby-Yao manifolds compactifications of the superstring. Duality symmetry was first described in the context of toroidal compactifications [1]. For the simplest case of single compactified dimension of radius R , the entire physics of interacting theory is left unchanged under the replacement $R \rightarrow \alpha/R$ provided one also transforms the dilaton field $\phi \rightarrow \phi - \ln(R/\sqrt{\alpha})$ [2]. This simple case can be generalized to arbitrary toroidal compactifications [3]. The T-duality symmetry was later extended to the case of nonflat conformal backgrounds possessing some abelian isometry (abelian T-duality) in [4].

Of more recent history is the notion of non-abelian duality [5, 6]. The basic idea of [5] is to consider a conformal field theory with non-abelian symmetry group. The non-abelian duality did miss a lot of features characteristic to the abelian duality. For example the non-abelian T-duality transformation of the isometric σ - model on a group manifold G gives non-isometric σ - model on its Lie algebra [5, 7, 8, 9]. As a result, it was not known how to perform the inverse duality transformation to get back to the original model. Indeed, while the original model on G was isometric, which was believed to be an essential condition for performing a duality transformation, the dual one did not possess the G - isometry.

A solution of this problem was proposed recently in [10] where it was argued that the two theories are dual to each other from the point of view of the so called Poisson-

Lie T-duality. In [10] a large class of new dual pairs of σ -models associated with each Drinfeld double [11] was constructed. The main idea of the approach is to replace the requirement of isometry by a weaker condition which is the Poisson-Lie symmetry of the theory. This generalized duality is associated with two groups forming a Drinfeld double and the duality transformation exchanges their roles.

The discussion in [10] was quite general. In order to apply Poisson-Lie T-duality in superstring theory one would like to know if there are dual pairs of conformal and superconformal σ -models. In particular, it would be interesting to construct mutually dual pairs of N=2 superconformal field theories.

The simple example of dual pair of conformal field theories associated with the $O(2,2)$ Drinfeld double was presented in work [12]. The supersymmetric generalization of Poisson-Lie T-duality was considered in [13]. The present note is devoted to the construction of dual pairs of N=2 superconformal WZNW models. In particular, after brief review of the Manin triple construction of N=2 superconformal WZNW models in the section 1, we will show in the section 2, that the classical N=2 superconformal WZNW models possess very natural Poisson-Lie symmetry which we will use to construct Poisson-Lie T-dual σ -models. In section 3 we will apply the results of section 2 to N=2 superconformal WZNW model associated with the Manin triple $(sl(2, R) \oplus R, b_+, b_-)$, where b_{\pm} are the Borel subalgebras of $sl(2, R)$ and construct its Poisson-Lie dual σ -model.

1. The classical N=1 superconformal WZNW model.

In this section we briefly review a supersymmetric WZNW (SWZNW) models using superfield formalism [14] and formulate conditions that a Lie group should satisfy in order for its SWZNW model to possess extended supersymmetry.

We parametrize superworld sheet introducing the light cone coordinates x_{\pm} , and grassman coordinates Θ_{\pm} . The generators of supersymmetry and covariant derivatives are

$$Q_{\mp} = \frac{\partial}{\partial \Theta_{\pm}} + \imath \Theta_{\pm} \partial_{\mp}, \quad D_{\mp} = \frac{\partial}{\partial \Theta_{\pm}} - \imath \Theta_{\pm} \partial_{\mp}. \quad (1)$$

They satisfy the relations

$$\{D_{\pm}, D_{\pm}\} = -\{Q_{\pm}, Q_{\pm}\} = -\imath 2 \partial_{\pm}, \quad \{D_{\pm}, D_{\mp}\} = \{Q_{\pm}, Q_{\mp}\} = \{Q, D\} = 0, \quad (2)$$

where the brackets $\{, \}$ denote the anticommutator. The superfield of N=1 supersymmetric WZNW model

$$G = g + \imath \Theta_- \psi_+ + \imath \Theta_+ \psi_- + \imath \Theta_- \Theta_+ F \quad (3)$$

takes values in a Lie group \mathbf{G} . We will assume that its Lie algebra \mathfrak{g} is endowed with ad-invariant nondegenerate inner product \langle, \rangle .

The inverse group element G^{-1} is defined from

$$G^{-1} G = 1 \quad (4)$$

and has the decomposition

$$G^{-1} = g^{-1} - \imath \Theta_- g^{-1} \psi_+ g^{-1} - \imath \Theta_+ g^{-1} \psi_- g^{-1} - \imath \Theta_- \Theta_+ g^{-1} (F + \psi_- g^{-1} \psi_+ - \psi_+ g^{-1} \psi_-) g^{-1} \quad (5)$$

For physical reasons one has to demand the group \mathbf{G} is a real manifold. Therefore it is convenient to consider \mathbf{G} as a subgroup in the group of real or unitary matrixes i.e. one has to impose the following conditons on the matrix elements of the superfield G :

$$\bar{g}^{mn} = g^{mn}, \quad \bar{\psi}_{\pm}^{mn} = \psi_{\pm}^{mn}, \quad \bar{F}^{mn} = F^{mn} \quad (6)$$

or

$$\bar{g}^{mn} = (g^{-1})^{nm}, \quad \bar{\psi}_{\pm}^{mn} = (\psi_{\pm}^{-1})^{nm}, \quad \bar{F}^{mn} = (F^{-1})^{nm}, \quad (7)$$

where we have used the following notations

$$\psi_{\pm}^{-1} = -g^{-1}\psi_{\pm}g^{-1}, \quad F^{-1} = -g^{-1}(F + \psi_{-}g^{-1}\psi_{+} - \psi_{+}g^{-1}\psi_{-})g^{-1}. \quad (8)$$

In the following we will assume that the superfield G satisfy (6) i.e. the Lie group \mathbf{G} is a subgroup of the group of nondegenerate real matrixes.

The action of N=1 SWZNW model is given by

$$S_{swz} = \int d^2x d^2\Theta (< R_{+}, R_{-} >) - \int d^2x d^2\Theta dt < G^{-1} \frac{\partial G}{\partial t}, \{R_{-}, R_{+}\} >, \quad (9)$$

where

$$R_{\pm} = G^{-1} D_{\pm} G. \quad (10)$$

The classical equations of motion can be obtained by making a variation of (9):

$$\delta S_{swz} = \int d^2x d^2\Theta < G^{-1} \delta G, D_{-} R_{+} - D_{+} R_{-} - \{R_{-}, R_{+}\} > \quad (11)$$

Taking into account kinematic relation

$$D_{+} R_{-} + D_{-} R_{+} = -\{R_{+}, R_{-}\} \quad (12)$$

we obtain

$$D_{-} R_{+} = 0 \quad (13)$$

The action (9) is invariant under the super-Kac-Moody and N=1 superconformal transformations [14].

In the following we will use supersymmetric version of Polyakov-Wiegman formula [15]

$$S_{swz}[GH] = S_{swz}[G] + S_{swz}[H] + \int d^2x d^2\Theta < G^{-1} D_{+} G, D_{-} H H^{-1} >. \quad (14)$$

It can be prooved as in the non supersymmetric case.

In works [16, 17, 18] supersymmetric WZNW models which admit extended supersymmetry were studied and correspondence between extended supersymmetric WZNW models and finite-dimensional Manin triples was established in [17, 18]. By the definition [11], a Manin triple $(\mathbf{g}, \mathbf{g}_{+}, \mathbf{g}_{-})$ consists of a Lie algebra \mathbf{g} , with nondegenerate invariant inner product $<, >$ and isotropic Lie subalgebras \mathbf{g}_{\pm} such that $\mathbf{g} = \mathbf{g}_{+} \oplus \mathbf{g}_{-}$ as a vector space. The corresponding Sugawara construction of N=2 Virasoro superalgebra generators was given in [17, 18, 19].

To make a connection between Manin triple construction of [17, 18] and approach of [16] based on complex structures on Lie algebras the following comment is relevant.

Let \mathfrak{g} be a real Lie algebra and J be a complex structure on the vector space \mathfrak{g} . J is referred to as a complex structure on a Lie algebra \mathfrak{g} if J satisfies the equation

$$[Jx, Jy] - J[Jx, y] - J[x, Jy] = [x, y] \quad (15)$$

for any elements x, y from \mathfrak{g} . Suppose the existence of a nondegenerate invariant inner product \langle, \rangle on \mathfrak{g} so that the complex structure J is skew-symmetric with respect to \langle, \rangle . In this case it is not difficult to establish the correspondence between complex Manin triples and complex structures on Lie algebras. Namely, for each complex Manin triple $(\mathfrak{g}, \mathfrak{g}_+, \mathfrak{g}_-)$ exists a canonic complex structure on the Lie algebra \mathfrak{g} such that subalgebras \mathfrak{g}_\pm are its $\pm i$ eigenspaces. On the other hand, for each real Lie algebra \mathfrak{g} with nondegenerate invariant inner product and skew-symmetric complex structure J on this algebra one can consider the complexification $\mathfrak{g}^{\mathbb{C}}$ of \mathfrak{g} . Let \mathfrak{g}_\pm be $\pm i$ eigenspaces of J in algebra $\mathfrak{g}^{\mathbb{C}}$ then $(\mathfrak{g}, \mathfrak{g}_+, \mathfrak{g}_-)$ is a complex Manin triple. Moreover it can be proved [17] that there exists one-to-one correspondence between complex Manin triple endowed with a hermitian conjugation (involutive antiisomorphism) $\tau : \mathfrak{g}_\pm \rightarrow \mathfrak{g}_\mp$ and the real Lie algebra endowed with ad -invariant nondegenerate inner product \langle, \rangle and the complex structure J which is skew-symmetric with respect to \langle, \rangle . Therefore we can use this conjugation to extract a compact form from a complex Manin triple.

If a complex structure on a Lie algebra is fixed then it defines the second supersymmetry transformation [16].

In this paper we concentrate on N=2 SWZNW models based on real Manin triples. The case of N=2 SWZNW models on compact groups will be considered in near future.

2. Poisson-Lie T-duality in N=2 superconformal WZNW model.

In this section we will describe the construction of Poisson-Lie T-dual σ -models to N=2 SWZNW models.

For the description of the Poisson-Lie T-duality in N=2 SWZNW we need a Lie group version of Manin triple [20, 21, 22]. Let's fix some Manin triple $(\mathfrak{g}, \mathfrak{g}_+, \mathfrak{g}_-)$ and consider double Lie group $(\mathbf{G}, \mathbf{G}_+, \mathbf{G}_-)$ [21], where the exponential groups \mathbf{G} , \mathbf{G}_\pm correspond to Lie algebras \mathfrak{g} , \mathfrak{g}_\pm . Each element $g \in \mathbf{G}$ admits a decomposition

$$g = g_+ g_-^{-1} \quad (16)$$

For SWZNW model on the group \mathbf{G} we obtain from (16) the decomposition for the superfield (4)

$$G(z_+, z_-) = G_+(z_+, z_-) G_-^{-1}(z_+, z_-) \quad (17)$$

Due to (17), (14) and the definition of Manin triple we can rewrite the action (9) for this model in the following form

$$S_{swz} = - \int d^2 x d^2 \Theta \langle \rho_+^+, \rho_-^- \rangle, \quad (18)$$

where the superfields

$$\rho^\pm = G_\pm^{-1} D G_\pm \quad (19)$$

correspond to the right invariant 1-forms on the groups \mathbf{G}_\pm .

To generalize (16) we have to consider the set W of classes $\mathbf{G}_+ \backslash \mathbf{G} / \mathbf{G}_-$ and pick up a representative w for each class $[w] \in W$ [22]:

$$\mathbf{G} = \bigcup_{[w] \in W} \mathbf{G}_+ w \mathbf{G}_- = \bigcup_{[w] \in W} \mathbf{G}_w \quad (20)$$

This formula means that there is the natural action of complex group $\mathbf{G}_+ \times \mathbf{G}_-$ on \mathbf{G} , and the set W parametrizes $\mathbf{G}_+ \times \mathbf{G}_-$ -orbits \mathbf{G}_w .

The corresponding generalization for the action (18) is given by

$$S_{swz} = - \int d^2 x d^2 \Theta \langle \rho_+^+, w \rho_-^- w^{-1} \rangle \quad (21)$$

Following [10] we consider a variation of the action (21) for SWZNW model on the group \mathbf{G} under the right action $\mathbf{G}_+ \times \mathbf{G}_-$. Let us concentrate at first on the class of identity from (20).

$$\begin{aligned} \delta S_{swz} = & - \int d^2 x d^2 \Theta (\langle D_+ X^+, \rho_-^- \rangle - \langle D_- X^-, \rho_+^+ \rangle) + \\ & \int d^2 x d^2 \Theta (\langle X^+, \{\rho_+^+, \rho_-^-\} \rangle - \langle X^-, \{\rho_-^-, \rho_+^+\} \rangle), \end{aligned} \quad (22)$$

where $X^\pm = G_\pm^{-1} \delta G_\pm$. From (22) we obtain the Noetherian currents ρ_\pm^\pm which satisfy on extremals

$$\begin{aligned} D_+ \rho_-^- + \{\rho_-^-, \rho_+^+\}^- &= 0, \\ D_- \rho_+^+ + \{\rho_-^-, \rho_+^+\}^+ &= 0, \end{aligned} \quad (23)$$

where the brackets $\{, \}$ correspond to Lie brackets on \mathfrak{g} . These equations can be joint into zero curvature equation for F_{+-} - component of super stress tensor F_{MN}

$$F_{+-} = \{D_+ + \rho_+^+, D_- + \rho_-^-\} = 0 \quad (24)$$

Using standard arguments of super Lax construction [23] one can show that from (24) it follows that the connection is flat

$$F_{MN} = 0, \quad M, N = (+, -, +, -). \quad (25)$$

The equations (24) are the supersymmetric generalization of Poisson-Lie symmetry conditions [10]. Indeed, Noetherian currents ρ_+^+, ρ_-^- are generators of $\mathbf{G}_+ \times \mathbf{G}_-$ - action on \mathbf{G} , while the structure constants in (24) correspond to Lie algebra \mathfrak{g} which is Drinfeld's dual to $\mathfrak{g}_+ \oplus \mathfrak{g}_-$ [24]. Due to (25) we may associate to each extremal surface $(G_+(x_+, x_-, \Theta_+, \Theta_-), G_-(x_+, x_-, \Theta_+, \Theta_-))$ a mapping $G(x_+, x_-, \Theta_+, \Theta_-)$ from the super world sheet into the group \mathbf{G} such that

$$L_-^+ + \rho_-^+ = L_-^- + \rho_-^- = 0, \quad (26)$$

$$L_+^+ = L_+^- = 0, \quad (27)$$

where L^\pm are \mathfrak{g}_\pm -components of the current $L = DGG^{-1}$ and $G \in \mathbf{G}$.

One can rewrite (26), (27) as the equations in Drinfeld's double of \mathbf{G} . Let

$$\mathbf{D} = \mathbf{G} \times \mathbf{G} \quad (28)$$

For each pair of superfields $\Lambda = (G_1, G_2) \in \mathbf{D}$ there is the decomposition

$$\Lambda = (G_+, G_-)(G, G) = H\tilde{H}, \quad (29)$$

where $H = (G_+, G_-)$, $\tilde{H} = (G, G)$. The equations (26), (27) can be rewritten in the form

$$\ll D_{\pm}\Lambda\Lambda^{-1}, E^{\mp} \gg = 0, \quad (30)$$

where \ll, \gg is the natural inner product on Lie algebra of \mathbf{D} and the subspaces E^{\pm} are given by

$$E^+ = (0, \mathbf{g}), \quad E^- = (\mathbf{g}, 0) \quad (31)$$

The equations (30) with appropriate choice of mutually orthogonal subspaces E^{\pm} are the supersymmetric generalization of corresponding equations from [10]. In the case when the subspaces are chosen in general position (which corresponds to nondegeneracy of the bilinear form determining the Lagrangian of σ -model) one can use the construction [10] to obtain the action for Poisson-Lie T-dual σ -model. But in our case the subspaces (31) are not in general position and the bilinear form determining the Lagrangian of N=2 SWZNW model is singular as it is easy to see from (18). It makes impossible to apply straightforward construction [10]. Instead we will use the method developed in [12].

Let us suppose that instead of (29) we use the decomposition

$$\Lambda = \tilde{F}F, \quad (32)$$

where $F = (U_+, U_-)$, $\tilde{F} = (U, U)$. Taking into account (31) we rewrite (30) in the following form

$$R_+^+ + \lambda_+^+ = R_-^- + \lambda_-^- = 0, \quad (33)$$

$$R_-^+ = R_+^- = 0, \quad (34)$$

where R^{\pm} are \mathbf{g}_{\pm} -components of the current $R = U^{-1}DU$, $U \in \mathbf{G}$, $\lambda_{\pm}^{\pm} = D_{\pm}U_{\pm}U_{\pm}^{-1}$, $U_{\pm} \in \mathbf{G}_{\pm}$. In dual picture we should have an action for dual σ -model on the group \mathbf{G} and the action of \mathbf{G} on itself such that the Noetherian currents satisfy zero curvature equation for F_{+-} -component of super stress tensor taking values in Lie algebra $\mathbf{g}_+ \oplus \mathbf{g}_-$ which is Drinfeld's dual Lie algebra to \mathbf{g} . But in view of the constraint (34) the action of dual σ -model has to be contained corresponding Lagrange multipliers. Using the arguments of [12] we can write the action of dual σ -model in the following form

$$\tilde{S}_{swz} = - \int d^2x d^2\Theta (\langle \lambda_+^+, \lambda_-^- \rangle + \langle R_+, \lambda_- \rangle + \langle \lambda_+, R_- \rangle) \quad (35)$$

It is easy to see from (35) that the currents λ_-^+ , λ_+^- play the role of the Lagrange multipliers (with values in $\mathbf{g}_+ \oplus \mathbf{g}_-$). The corresponding equations of motion include apart from (33), (34) zero curvature equation

$$\tilde{F}_{+-} = \{D_+ - \lambda_+^+ + \lambda_+^-, D_- - \lambda_-^- + \lambda_-^+\} = 0, \quad (36)$$

where the brackets $\{, \}$ correspond to Lie brackets on $\mathfrak{g}_+ \oplus \mathfrak{g}_-$. Excluding from (35) all λ 's except the Lagrange multipliers we obtain

$$\tilde{S}_{swz} = - \int d^2x d^2\Theta (< R_+^+, R_-^- > + < R_+^-, \lambda_-^+ > + < \lambda_+^-, R_-^+ >) \quad (37)$$

Now we turn to the remainder adjacent classes from (20). The generalization of (24), (26), (27) is straightforward. In each class $[w]$ the Noetherian currents ρ_{w+}^+ and ρ_{w-}^- take values in the subspaces $\mathfrak{g}_+^w = w^{-1}\mathfrak{g}_+w \cup \mathfrak{g}_+$ and $\mathfrak{g}_-^w = w\mathfrak{g}_-w^{-1} \cup \mathfrak{g}_-$ correspondingly, while the constraints L_{w+}^- and L_{w-}^+ take values in the complements $\mathfrak{g} \setminus \mathfrak{g}_+^w$ and $\mathfrak{g} \setminus \mathfrak{g}_-^w$:

$$F_{w+-} = \{D_+ + \rho_{w+}^+, D_- + \rho_{w-}^-\} = 0 \quad (38)$$

$$L_{w\pm}^\pm + \rho_{w\pm}^\pm = 0, \quad (39)$$

$$L_{w\mp}^\pm = 0, \quad (40)$$

The arguments we have used to obtain (37) can be applied (with relevant modifications) to each class $[w]$. Taking into account (38), (39), (40) we obtain the generalization of (37)

$$\tilde{S}_{swz} = - \int d^2x d^2\Theta (< R_{w+}^+, R_{w-}^- > + < R_{w+}^-, \lambda_{w-}^+ > + < \lambda_{w+}^-, R_{w-}^+ >), \quad (41)$$

where R_{w+}^+ and R_{w-}^- take values in the same subspaces like the currents ρ_{w+}^+ and ρ_{w-}^- , R_{w+}^- and R_{w-}^+ take values in the complements $\mathfrak{g} \setminus \mathfrak{g}_+^w$ and $\mathfrak{g} \setminus \mathfrak{g}_-^w$ correspondingly.

3. Poisson-Lie T-dual σ -model to N=2 SWZNW model on $SL(2, \mathbf{R}) \times \mathbf{R}$.

The Lie algebra of the group $\mathbf{G} = SL(2, R) \times R$ has the basis

$$e_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad e_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad (42)$$

$$e^0 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \quad e^1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad (43)$$

Note that both sets of generators (42), (43) span the Borelian subalgebras \mathbf{b}_- , \mathbf{b}_+ correspondingly and they are maximally isotropic with respect to the non-degenerate invariant inner product defined by the brackets

$$< e_i, e^j > = \delta_i^j. \quad (44)$$

Hence we have Manin triple $(\mathfrak{g}, \mathbf{b}_+, \mathbf{b}_-)$. Let $\mathbf{B}_\pm = \exp(\mathbf{b}_\pm)$. The decomposition (20) is given by Bruhat decomposition

$$\mathbf{G} = \mathbf{G}_1 \bigcup \mathbf{G}_w, \quad (45)$$

where

$$\begin{aligned} \mathbf{G}_1 &= \mathbf{B}_+ \mathbf{B}_-, \\ \mathbf{G}_w &= \mathbf{B}_+ \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{B}_- \end{aligned} \quad (46)$$

In the class of identity we parametrize the element $g \in \mathbf{G}_1$ by the matrix

$$g = \begin{pmatrix} 1 & 0 \\ u & 1 \end{pmatrix} \begin{pmatrix} 1 & v^{-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \exp(a_+) & 0 \\ 0 & \exp(a_-) \end{pmatrix}. \quad (47)$$

The classical constraint $R_+^- = 0$ takes the form

$$\begin{aligned} D_+ a_+ - v^{-1} D_+ u &= 0, \\ D_+(u + v) &= 0, \end{aligned} \quad (48)$$

the classical constraint $R_-^+ = 0$ takes the form

$$\begin{aligned} D_- u &= 0, \\ D_- a_- + v^{-1} D_- u &= 0 \end{aligned} \quad (49)$$

Under these constraints, the remaining components of the currents are

$$\begin{aligned} R_+^+ &= -2D_+ \omega e^0 - \exp(2\phi) D_+ v e^1, \\ R_-^- &= 2D_- \omega e_0 - \exp(-2\phi) v^{-2} D_- v e_1, \end{aligned} \quad (50)$$

where we have introduced new variables

$$\phi = (a_+ - a_-)/2, \quad \omega = (a_+ + a_-)/2 \quad (51)$$

so that the action in (37) becomes

$$\tilde{S}_1 = \int d^2x d^2\Theta (-4D_+ \omega D_- \omega + D_+ \ln v D_- \ln v) \quad (52)$$

and thus describes two free scalar superfields ω and $\ln v$.

In the class [w] we have an appart from constraints (48), (49) additional constraints

$$\begin{aligned} \exp(a_+ - a_-) D_+ u &= 0, \\ \exp(a_- - a_+) v^{-2} D_- (u + v) &= 0. \end{aligned} \quad (53)$$

Under the constraints (48), (49), (53) the action in (41) becomes

$$\tilde{S}_w = - \int d^2x d^2\Theta 4D_+ \omega D_- \omega \quad (54)$$

and thus describes free scalar superfield ω .

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References

- [1] L. Brink, M. B. Green and J. H. Schwartz, *Nucl.Phys* **B198** (1982) 474; K.Kikkawa, M.Yamasaki, *Phys. Lett.* **B149** (1984) 357;
- [2] E. Alvarez and M. A. R. Osorio, *Phys. Rev.* **D40** (1989) 1150;
- [3] K. Narain, H. Sarmadi and E. Witten, *Nucl.Phys* **B279** (1987) 369;
- [4] T. H. Busher, *Phys. Lett.* **B194** (1987) 51; *Phys. Lett.* **B201** (1988) 466;
- [5] X. De la Ossa and F. Quevedo, *Nucl. Phys.* **B403** (1993) 377;
- [6] A. Giveon and M. Rocek, *Nucl. Phys.* **B421** (1994) 173; E. Alvarez, L. Alvarez-Gaume and Y. Lozano, *Phys. Lett.* **B336** (1994) 183; A. Giveon, E. Rabinovichi and G. Veneziano, *Nucl. Phys.* **B322** (1989) 167;
- [7] B. E. Fridling and A. Jevicki, *Phys. Lett.* **B134** (1984) 70;
- [8] E. S. Fradkin and A. A. Tseytlin, *Ann. Phys.* **162** (1985) 31;
- [9] T. Curtright and C. Zachos, *Phys. Rev.* **D49** (1994) 5408; T. Curtright and C. Zachos, *Phys. Rev.* **D52** (1995) R573;
- [10] C. Klimcik and P. Severa, *Phys. Lett.* **B351** (1995) 455; C. Klimcik and P. Severa, Poisson-Lie T-duality and Loop groups of Drinfeld Doubles, *CERN-TH/95-330* hep-th/9512040;
- [11] V. G. Drinfeld, Quantum groups, *Proc. Int. Cong. Math., Berkley, Calif.* (1986) 798.
- [12] A. Yu. Alekseev, C. Klimcik and A. A. Tseytlin, Quantum Poisson-Lie T-duality and WZNW model, *CERN-TH/95251* hep-th/9509123
- [13] K. Sfetsos, Poisson-Lie T-duality and supersymmetry, *THU-96/38* hep-th/9611199
- [14] P. DI Vecchia, V. G. Knizhnik, J. L. Petersen and P. Rossi, *Nucl.Phys* **B253** (1985) 701;
- [15] A. Polyakov and P. Wiegmann, *Phys. Lett.* **B131** (1983) 121;
- [16] Ph. Spindel, A. Sevrin, W. Troost and A. van Proeyen, *Nucl. Phys.* **B308** (1988) 662; *Nucl. Phys.* **B311** (1988/89) 465;
- [17] S. E. Parkhomenko, *Zh. Eksp. Teor. Fiz.* **102** (July 1992) 3-7.
- [18] S. E. Parkhomenko, *Mod. Phys. Lett.* **A11** (1996) 445;
- [19] E. Getzler, Manin Pairs and topological Field Theory, *MIT-preprint* (1994).
- [20] M. A. Semenov-Tian-Shansky, Dressing Transformations and Poisson-Group Actions, *RIMS, Kyoto Univ.* **21** (1985) 1237.
- [21] J.-H. Lu and A. Weinstein, *J. Diff. Geom.* **V31** (1990) 501.

- [22] A. Yu. Alekseev and A. Z. Malkhin, *Commun. Math. Phys.* **V162** (1994) 147.
- [23] J. Evans and T. Hollowood, *Nucl. Phys.* **B352** (1991) 723;
- [24] V. G. Drinfeld, *DAN SSSR* **V262** (1983) 285.